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Chaotic behaviour of Heart rate variability through discrete dynamical model

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Abstract-This work deals about non-linear dynamical model for biological signals such as electroencephalogram (EEG), heart rate, breathing pattern, blood pressure and peripheral blood flow. So far, we use the spectral analysis mainly for a non-linear oscillating phenomenon and for the separation of the oscillatory components of the wave. In this work we use the chaotic theory and analytical procedure for biological signals such as the electroencephalogram (EEG). Moreover the HRV showed a chaos-like property. We determine a chaotic property from the attractor embedded in the 2-dimensional space

Index Terms - Chaos, non-linear dynamics, heart rate variability.

1. Introduction:

Heart rate variability (HRV) refers to the beat-to-beat alterations in heart rate. Under resting conditions, the ECG of healthy individuals exhibits periodic variation in R-R intervals. This rhythmic phenomenon, known as respiratory sinus arrhythmia (RSA), fluctuates with the phase of respiration-cardio-acceleration during inspiration and cardio-deceleration during expiration. RSA is predominantly mediated by respiratory gating of parasympathetic efferent activity to the heart: vagal efferent traffic to the sinus node occurs primarily in phase with expiration and is absent or attenuated during inspiration. Atropine abolishes RSA.

Reduced HRV has thus been used as a marker of reduced vagal activity. However, because HRV is a cardiac measure derived from the ECG, it is not possible to distinguish reduced central vagal activity (in the vagal centers of the brain) from reduced peripheral activity (the contribution of the target organ — the sinus node — or the afferent/efferent pathways conducting the neural impulses to/from the brain).

Although our understanding of the meaning of HRV is far from complete, it seems to be a marker of both dynamic and cumulative load. As a dynamic marker of load, HRV appears to be sensitive and responsive to acute stress. Under laboratory conditions, mental load — including making complex decisions, and public speech tasks — have been shown to lower HRV. As a marker of cumulative wear and tear, HRV has also been shown to decline with the aging process. Although resting heart rate does not change significantly with advancing age, there is a decline in HRV, which has been attributed to a decrease in efferent vagal tone and reduced beta-adrenergic responsiveness. By contrast, regular physical activity (which slows down the aging process) has been shown to raise.

2. HRV, presumably by increasing vagal tone

In short, HRV appears to be a marker of two processes, relevant to the conceptualization of allostatic load: (1) frequent activation (short term dips in HRV in response to acute stress); and (2) inadequate response (long-term vagal withdrawal, resulting in the over-activity of the counter-regulatory system — in this case, the sympathetic control of cardiac rhythm).

3. To Measure HRV

Originally, HRV was assessed manually from calculation of the mean R-R interval and its standard deviation measured on short-term (e.g., 5 minute) electrocardiograms. The smaller the standard deviation in R-R intervals, the lower is the HRV. To date, over 26 different types of arithmetic manipulations of R-R intervals have been used in the literature to represent HRV. Examples include: the standard deviations of the normal mean R-R interval obtained from successive 5-minute periods over 24-hour Holter recordings (called the SDANN index); the number of instances per hour in which two consecutive R-R intervals differ by more than 50 msec over 24-hours (called the pNN50 index); the root-mean square of the difference of successive R-R intervals (the rMSSD index); the difference between the shortest R-R interval during inspiration and the longest during expiration (called the MAX-MIN, or peak-valley quantification of HRV); and the base of the triangular area under the main peak of the R-R interval frequency distribution diagram obtained from 24-hour recording; and so on. So far, experimental and simulation data appear to indicate that the various methods of expressing HRV are largely equivalent, and there is no evidence that any one method is superior to another, provided measurement windows are 5 minutes or longer.

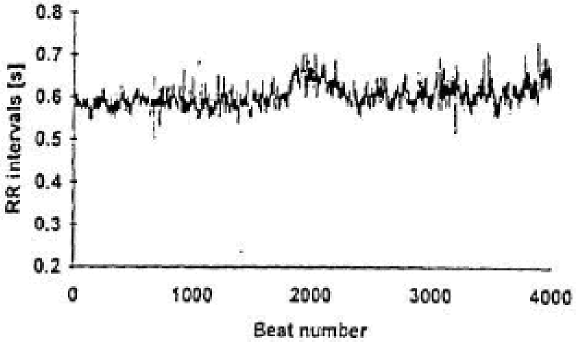

Figure 01- Tachogram

In contrast to the so-called time domain measures of HRV cited above, recent developments in microprocessor technology has enabled the calculation of frequency measures based on mathematical manipulations performed on the same ECG-derived data. Frequency measures involve the spectral analysis of HRV. Briefly, R-R interval data are represented on a tachogram (Figure 01), in which the y-axis plots the R-R intervals, and the x-axis the total number of beats. Spectral analysis of the tachogram transforms the signal from time to frequency on the x-axis, by representing the signal as a combination of sine and cosine waves, with different amplitudes (Figure 02).

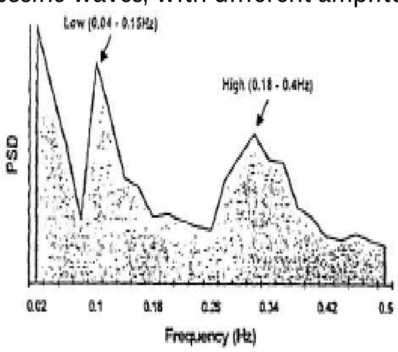

Figure 02 - Power spectrum of HRV
(PSD = power spectral density)

The approach uses Fourier transforms. The HRV spectrum contains two major components: the high frequency (0.18-0.4 Hz) component, which is synchronous with respiration and is identical to RSA. The second is a low frequency (0.04 to 0.15 Hz) component that appears to be mediated by both the vagus and cardiac sympathetic nerves. The power of spectral components is the area below the relevant frequencies presented in absolute units (square milliseconds). The total power of a signal, integrated over all frequencies, is equal to the variance of the entire signal. Some investigators have used the ratio of the low-to-high frequency spectra as an index of parasympathetic-sympathetic balance; however, this remains controversial because of our lack of complete understanding of the low frequency component (which seems to be affected by centrally generated brainstem rhythms, baroreceptor feedback influences, as well as both sympathetic and vagal input).

As a measure of vagal activity, spectral analysis of the high-frequency component probably offers no additional information over time-domain measures of RSA. On the other hand, the meaning and utility of the low frequency component deserves further investigation.

In sum, the analysis of HRV (whether by time-domain or spectral approaches) offers a non-invasive method of evaluating vagal input into cardiac rhythm. The measurement of HRV is becoming increasingly standardized (e.g., see report of the Task Force of the European Society of Cardiology, 1996). Although, the assessment of HRV requires electro physiologic expertise, the equipment is not prohibitively expensive, requiring only ECG equipment, microprocessors, and relevant software for carrying out Fourier analyses.

4**. HRV - predict disease**:

The major reason for the interest in measuring HRV stems from its ability to predict survival after heart attack. Over half a dozen prospective studies have shown that reduced HRV predicts sudden death in patients with MI, independent of other prognostic indicators such as ejection fraction. Reduced HRV appears to be a marker of fatal ventricular arrhythmia. Moreover, a small number of studies have begun to suggest that reduced HRV may predict risk of survival even among individuals free of CHD.

5**. Materials and Methods**

Fifty patients whose heart rate variability (ECG) were taken for my study. Some of the patients have chronic metabolic or cardiovascular disorders. Some have no cardiovascular diseases.

5.1 **Data Recording**:

The patients entered the measurement room and equipped the electrodes for electrocardiogram (ECG), after a sufficient rest on the sitting posture. An ECG was measured with a standard bipolar lead from the chest by using a bioamplifier and stored simultaneously on a data recorder for the subsequent analysis. During the measurements, the patients kept a spontaneous breathing and relaxation.

5.2 **Data Processing**:

After the experiments an ECG data was transferred to the computer for primary data file. We considered the Lead II for our study. The data is set for the threshold time at 0.5 seconds. Following the calculation of the average from the Lead II values the standard deviation of the HR time series data is calculated.

5.3 **Lyapunov Spectrum**:

The Lyapunov exponent λ is used to measure the exponential rate of divergence of neighbouring trajectories. It is expressed by the formula

$$\lambda = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} \log\left|e_i^{'}(t)\right| \qquad i = 1,2,3,4,5..........$$

With respect to the estimated Lyapunov spectrum, if the maximum Lyapunov exponent is positive value, it will have an orbital instability and then it is identified as chaos.

5.4 **Fractal Dimension**:

In this study it is identified that a point is zero dimension, a straight line is one dimension, a plane is two dimensions and a space is three dimensions. Such dimensions are integers.

Approaches for the determination of this dimension were introduced, in the present study the Lyapunov dimension was used as the fractal dimensions.

5.5**. ANALYSIS**

ECG is collected from 50 patients with normal procedure and the data is tabulated below.

Patient. No.	Lead-II(1)	Lead-II(2)	Lead-II(3)	$\bar{X}$	Ly.Exp.(λ)	S.D(σ)
1	60.00	60.00	57.69	59.23	1.81	9.57
2	75.00	71.43	71.43	72.62	1.82	3.08
3	60.00	60.00	62.50	60.83	1.88	8.60
4	62.50	65.22	65.22	64.31	1.95	6.68
5	115.00	107.00	107.00	109.67	2.02	3.81
6	107.00	100.00	100.00	102.33	1.94	1.48
7	100.00	100.00	100.00	100.00	1.91	0.97
8	65.00	63.00	68.00	65.33	1.85	6.17
9	83.00	79.00	83.00	81.67	1.88	0.72
10	65.00	65.00	65.00	65.00	1.90	6.33
11	83.00	79.00	79.00	80.33	1.94	0.97
12	100.00	93.75	100.00	97.92	2.02	0.60
13	83.33	93.75	83.33	86.81	1.95	0.11
14	136.36	136.36	125.00	132.58	2.02	17.99
15	60.00	62.50	60.00	60.83	1.95	8.60
16	150.00	136.36	150.00	145.45	2.00	30.57
17	83.33	83.33	78.95	81.87	1.94	0.69
18	83.33	88.24	83.33	84.97	1.95	0.27
19	75.00	93.75	125.00	97.92	1.98	0.60
20	88.24	78.95	88.24	85.14	1.99	0.25
21	115.38	115.38	75.00	101.92	2.01	1.38
22	88.24	125.00	115.38	109.54	1.99	3.76
23	78.95	93.75	125.00	99.23	2.01	0.82
24	88.24	83.33	78.95	83.51	1.92	0.44
25	136.36	125.00	136.36	132.58	1.87	17.99
26	50.00	53.57	53.57	52.38	1.77	14.12
27	55.56	55.56	60.00	57.04	1.77	10.97
28	68.18	65.22	65.22	66.21	1.80	5.74
29	57.69	55.56	53.57	55.61	1.77	11.94
30	62.50	75.00	71.43	69.64	1.76	4.21
31	51.72	53.57	48.39	51.23	1.80	15.16
32	55.56	50.00	53.57	53.04	1.95	13.78
33	100.00	100.00	83.33	94.44	2.10	0.18
34	136.36	136.36	150.00	140.91	2.15	25.76
35	150.00	136.36	150.00	145.45	2.06	30.57
36	136.36	136.36	150.00	140.91	2.04	25.76
37	75.00	75.00	75.00	75.00	2.02	2.30
38	125.00	125.00	115.38	121.79	2.11	10.00
39	115.38	125.00	125.00	121.79	1.99	10.00
40	150.00	136.36	150.00	145.45	1.96	30.57
41	53.57	51.72	55.56	53.62	1.93	13.35
42	100.00	78.95	115.38	98.11	2.00	0.63
43	125.00	136.36	100.00	120.45	1.91	9.17
44	88.24	78.95	83.33	83.51	1.91	0.44
45	60.00	51.72	53.57	55.10	1.91	12.29
46	115.38	125.00	115.38	118.59	1.91	8.08
47	100.00	78.95	75.00	84.65	1.79	0.30
48	55.56	51.72	53.57	53.62	1.86	13.35
49	50.00	50.00	51.72	50.57	1.94	15.67
50	125.00	136.36	150.00	137.12	2.11	22.05

Table -1

For this data Logistic Map is fitted by Numerical technique. It is obtained as approximately 2. This implies ECG data shows Low Dimensional Chaos. Also a two dimensional 2D Phase Space graph is constucted by x_n Vs $x_{n+\tau}$

The choosing of τ is important for calculating correlation dimension. We can conveniently choose the optimal τ from the Phase Space Graph. If τ is too small, x_n is close to $x_{n+\tau}$ and the attractor is compressed to the vicinity of the diagonal in Phase Space so that it cannot be completely intended fig-01. If τ is too large, the attraction will fold and the Phase Space deforms fig-02. & fig-03. We believe that τ = 3 is optimal.

5.6 **Figures**

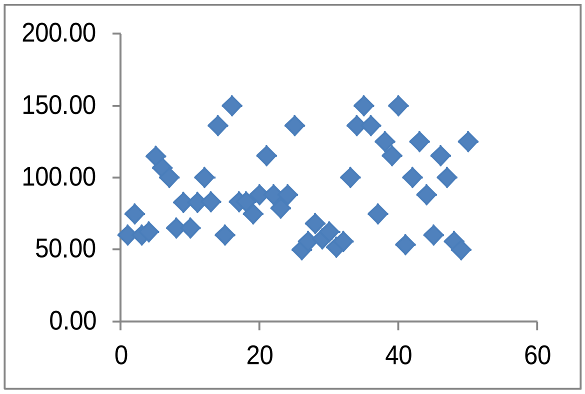

Figure 01
Phase space graph between X_n and X_{n+1}

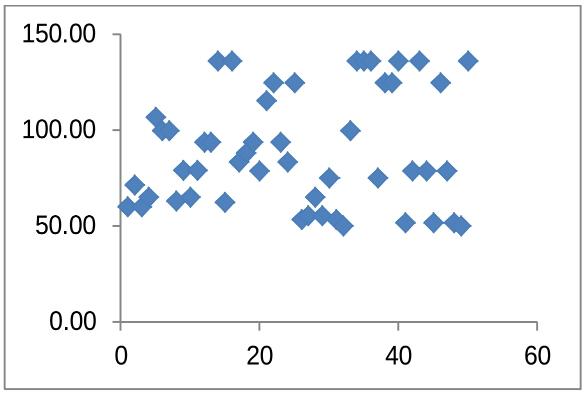

Figure 02
Phase space graph between X_n and X_{n+2}

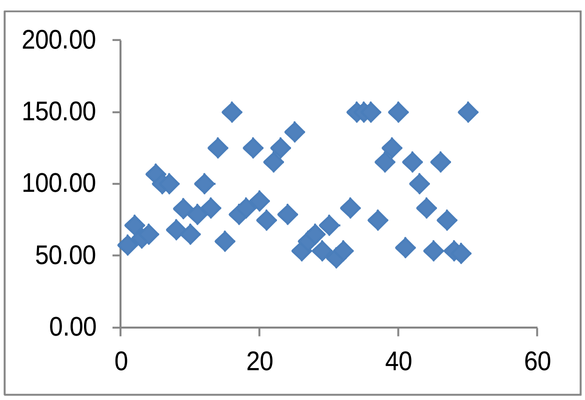

Figure 03
Phase space graph between X_n and X_{n+3}

5.7. **Correlation Coefficient**:

Correlation Coefficient is obtained between Lyapunov Exponent τ and Standard Deviation σ using Karl Pearson's coefficient of correlation (ρ).

ρ is obtained as 0.191

This implies as expected there is a low degree of positive correlation between Lyapunov Exponent and Standard Deviation of HRV.

6. **Findings:**

The major findings of the present study are as follows.

1.There exists a low dimensional chaos in the HRV. This is inferred by finding l in the Logistic equation.

$$y_{t+1} = \lambda\, y_t(1-y_t)$$

2. HRV shows non-linear characteristics in their behaviour.
3. There is a low degree of positive correlation between Lyapunov Exponent and Standard Deviation of HRV.

7. **Conclusion:**

The study of Chaos is related to so many fields. We compared Phase Space Diagram for i = 1,2 & 3. We have studied ECG readings of 50 patients with regard to Non-Linear study. Therefore it is not appropriate to describe the Human Heart, a Non-Linear Dynamic System with complex structure by only dimension. This study can be extended by finding various dimensions. There are various tools to study Non-Linear Dynamics structure such as Hurst Component, ARIMA models etc.. In fact ECG signals provide valuable diagnostic information about location of ischmia or infarcetian. This study can be made elaborate by applying various tools in Chaotic Dynamics and Non-Linear Dynamics.

8. **References**:

[1] Hames T. Sandefur, "Discrete Dynamical system , Theory and Application" .

[2] William J. Banmol and Jess Benhbib "Chaos, Significance, Mechanism and Economic Application, Journal of Economic perspectives, vol 3, No. 1, 1989, pp 77 - 105.

[3] Hans. A. Lauwerier, "One-dimensional Iterative Map" in Arun .V. Holder, ed, chaos Princetan University press, Princetan, NJ, 1986.

[4] Grassberger and Procacci, "Measuring the strangeness of strange attractor, Physica D, 9:1983, pp 189 - 208.

[5] Kawachi I, Colditz GA, Ascherio A, Rimm EB, Giovannucci E, Stampfer MJ, Willett WC, Prospective study of phobic anxiety and risk of coronary heart disease, Circulation 1994 a; 89 : 1992 - 1997.

[6] Offerhaus RE, Heart Rate Variability in Psychiatry in RJ Kitney, Rompelman O (eds), The study of

Heart Rate Variability, Oxford, Oxford University Press, 1980: pp 225 - 238.

[7] Yeragani VK, Balon R, Pohl R, Ramesh C, Glitz D, Weinberg P, Merlos B, Decreasd R-R variance in panic disorder patients, Acta Psychiatr Scand 1990; 81: 554 - 559.

[8] Yeragani VK, Pohl R, Berger R, Balon R, Ramesh C, Glitz D, Srinivasan K, Weinberg P, Decreased heart rate variablility in panic disorder patients: a study of power - spectral analysis of heart rate. Psychiatry Res 1993; 46 : pp 89 - 103.

[9] Haines AP, Imeson JD, Meade TW, Phobic anxiety and ischaemic Heart diseases. Br Med J 1987; vol 295 : pp 297-299.

[10] Kawachi I, sparrow D, Vokonas PS, Weiss ST, Symptoms of anxiety and risk of coronary heart disease : The Normative Aging Study, Circulation 1994B; vol 90 : pp 2225 - 2229.

8. Author Bibliography

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